

SUPPLEMENTARY FILES FOR “A CONSTRAINED MANY-OBJECTIVE OPTIMIZATION EVOLUTIONARY ALGORITHM WITH ENHANCED MATING AND ENVIRONMENTAL SELECTIONS”

A. Parameter Settings

TABLE S-I

PARAMETER SETTINGS FOR THE 10 METHODS ON DTLZ BENCHMARK, WHERE N IS THE POPULATION SIZE AND E_{\max} IS THE MAXIMUM NUMBER OF FUNCTION EVALUATIONS.

m	N	E_{\max}
3	91	30,000
5	126	50,000
8	156	80,000
10	220	100,000
15	240	120,000

B. Performance Indicators

In this work, the two widely used indicators, inverted generational distance (IGD) and hypervolume (HV), are as follows:

- *IGD*: Represents the average distance from each reference point to the nearest individual. Suppose \mathcal{S}^* is a set of uniformly distributed points on the true PF and \mathcal{S} is the solution set. The IGD value is calculated as

$$\text{IGD}(\mathcal{S}^*, \mathcal{S}) = \frac{\sum_{\mathbf{x} \in \mathcal{S}^*} d_{\text{nn}}(\mathbf{x}, \mathcal{S})}{|\mathcal{S}^*|},$$

where $d_{\text{nn}}(\mathbf{x}, \mathcal{S})$ is the Euclidean distance between \mathbf{x} and its nearest neighbor in \mathcal{S} . The smaller the IGD value, the better the performance of an algorithm.

- *HV*: Measures the volume or hypervolume of the objective space enclosed by the obtained solution set and the predefined reference point \mathbf{z}^r ; HV of a solution set \mathcal{S} is formulated as

$$\text{HV}(\mathcal{S}) = \text{VOL} \left(\bigcup [z_1, z_1^r] \times \cdots \times [z_m, z_m^r] \right),$$

where VOL indicates the Lebesgue measure. The algorithm with a larger HV value indicates better performance.

C. Supplementary Results

TABLE S-II

STATISTICAL RESULTS OF HV ON C-DTLZ AND DC-DTLZ OBTAINED BY CMME AND CMAOEs IN COMPARISON, THE BEST VALUES OF EACH ROW ARE HIGHLIGHTED.

Table with 11 columns: Problem, m, NSGA-II-ToR, TOP, CCMO, NSGA-III, I-DBEA, PPS, C-TAEA, TIG2-E, DCNSGA-III, CMME. Rows include C1-DTLZ1, C1-DTLZ3, C2-DTLZ2, C3-DTLZ4, DC1-DTLZ1, DC1-DTLZ3, DC2-DTLZ1, DC2-DTLZ3, DC3-DTLZ1, DC3-DTLZ3. Each cell contains numerical values or null, with best values in bold.

TABLE S-III

STATISTICAL RESULTS OF IGD OBTAINED BY CMME AND CMAOEs IN COMPARISON, THE BEST VALUES OF EACH ROW ARE HIGHLIGHTED.

Table with 11 columns: Problem, m, NSGA-II-ToR, TOP, CCMO, NSGA-III, I-DBEA, PPS, C-TAEA, TIG2-E, DCNSGA-III, CMME. Rows include C1-DTLZ1, C1-DTLZ3, C2-DTLZ2, C3-DTLZ4, DC1-DTLZ1, DC1-DTLZ3, DC2-DTLZ1, DC2-DTLZ3, DC3-DTLZ1, DC3-DTLZ3. Each cell contains numerical values or null, with best values in bold.

TABLE S-VI

STATISTICAL RESULTS OF HV AND IGD OBTAINED BY CMME AND ITS VARIATIONS, THE BEST VALUES OF EACH ROW ARE HIGHLIGHTED.

Table with columns: Problem, m, HV (CMME-1, CMME-2, CMME-3, CMME-4), CMME, IGD (CMME-1, CMME-2, CMME-3, CMME-4), CMME. Rows include CI-DTLZ1, CI-DTLZ2, C2-DTLZ2, C3-DTLZ4, DC1-DTLZ1, DC1-DTLZ3, DC2-DTLZ1, DC2-DTLZ3, DC3-DTLZ1, DC3-DTLZ3.

TABLE S-VII

SUMMARY OF THE WILCOXON TEST ON HV. HEREINAFTER, “*” MEANS THE METHOD IN THE ROW OUTPERFORMS THE METHOD IN THE COLUMN. “o” INDICATES THE METHOD IN THE COLUMN OUTPERFORMS THE METHOD IN THE ROW. UPPER DIAGONAL OF LEVEL SIGNIFICANCE AT alpha = 0.9, AND LOWER DIAGONAL LEVEL OF SIGNIFICANCE AT alpha = 0.95.

Comparison matrix for HV test showing performance between algorithms: NSGA-II-ToR (1), ToP (2), CCMO (3), NSGA-III (4), I-DBEA (5), PPS (6), C-TAEA (7), TiGE-2 (8), DCNSGA-III (9), CMME (10).

TABLE S-VIII

SUMMARY OF THE WILCOXON TEST ON THE IGD METRIC.

Comparison matrix for IGD test showing performance between algorithms: NSGA-II-ToR (1), ToP (2), CCMO (3), NSGA-III (4), I-DBEA (5), PPS (6), C-TAEA (7), TiGE-2 (8), DCNSGA-III (9), CMME (10).

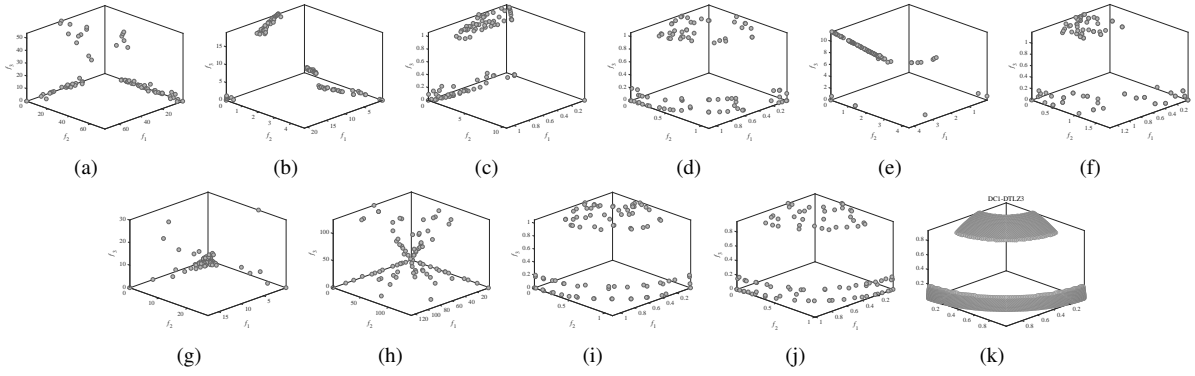


Fig. S-2. Feasible and non-dominated solutions with the median HV value among 30 runs on 3-objective DC1-DTLZ3. (a) NSGA-II-ToR, (b) ToP, (c) CCMO, (d) NSGA-III, (e) I-DBEA, (f) PPS, (g) C-TAEA, (h) TiGE-2, (i) DCNSGA-III, (j) CMME, and (k) true PF.

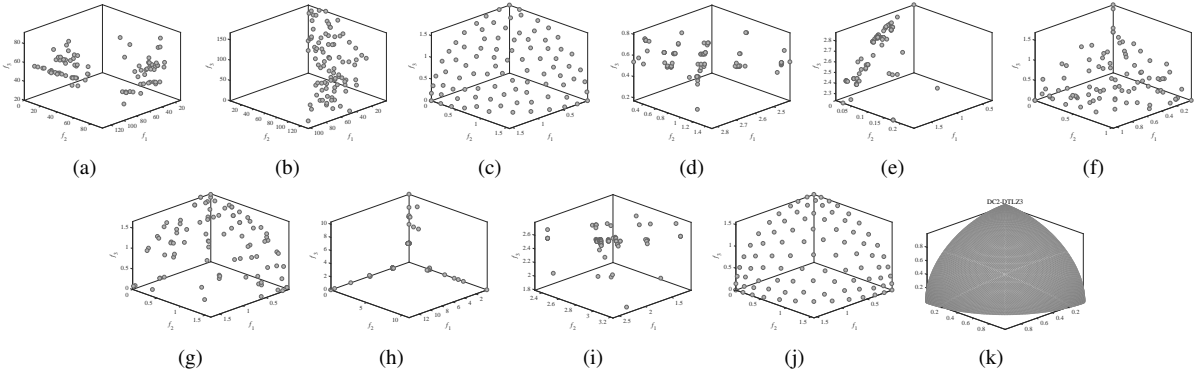


Fig. S-3. Feasible and non-dominated solutions with the median HV value among 30 runs on 3-objective DC2-DTLZ3. (a) NSGA-II-ToR, (b) ToP, (c) CCMO, (d) NSGA-III, (e) I-DBEA, (f) PPS, (g) C-TAEA, (h) TiGE-2, (i) DCNSGA-III, (j) CMME, and (k) true PF.

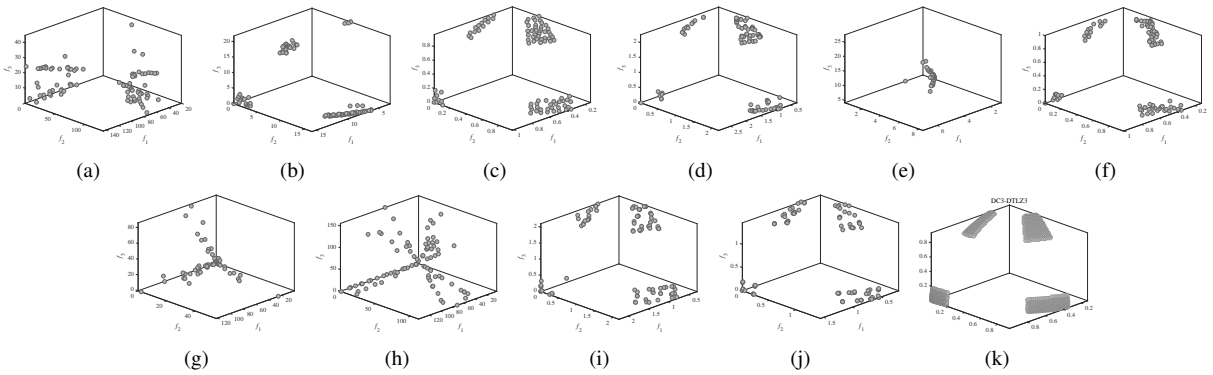


Fig. S-4. Feasible and non-dominated solutions with the median HV value among 30 runs on 3-objective DC3-DTLZ3. (a) NSGA-II-ToR, (b) ToP, (c) CCMO, (d) NSGA-III, (e) I-DBEA, (f) PPS, (g) C-TAEA, (h) TiGE-2, (i) DCNSGA-III, (j) CMME, and (k) true PF.

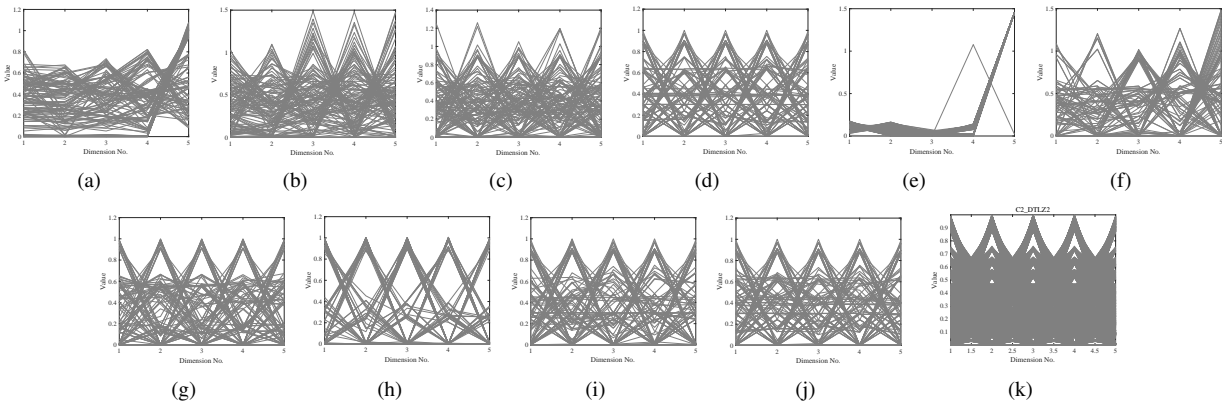


Fig. S-5. Feasible and non-dominated solutions with the median HV value among 30 runs on 5-objective C2-DTLZ2. (a) NSGA-II-ToR, (b) ToP, (c) CCMO, (d) NSGA-III, (e) I-DBEA, (f) PPS, (g) C-TAEA, (h) TiGE-2, (i) DCNSGA-III, (j) CMME, and (k) true PF.

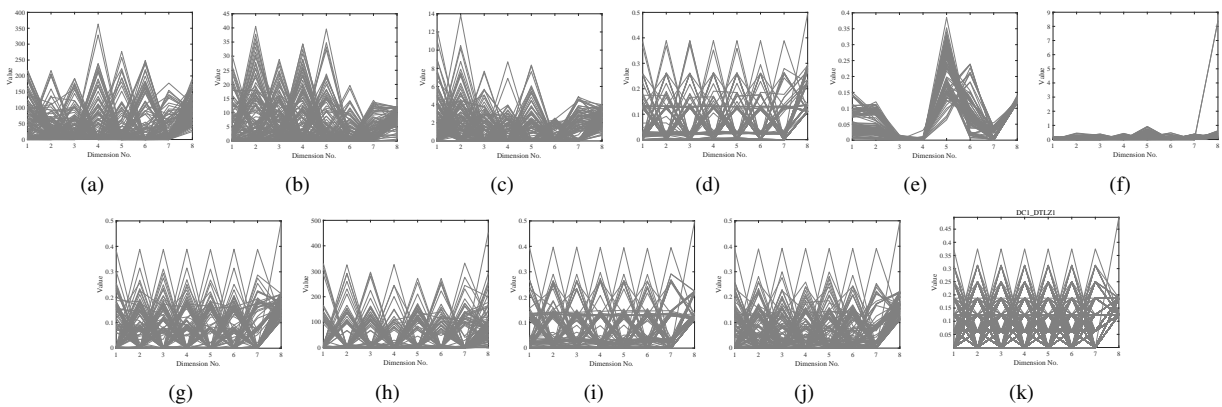


Fig. S-6. Feasible and non-dominated solutions with the median HV value among 30 runs on 8-objective DC1-DTLZ1. (a) NSGA-II-ToR, (b) ToP, (c) CCMO, (d) NSGA-III, (e) I-DBEA, (f) PPS, (g) C-TAEA, (h) TiGE-2, (i) DCNSGA-III, (j) CMME, and (k) true PF.

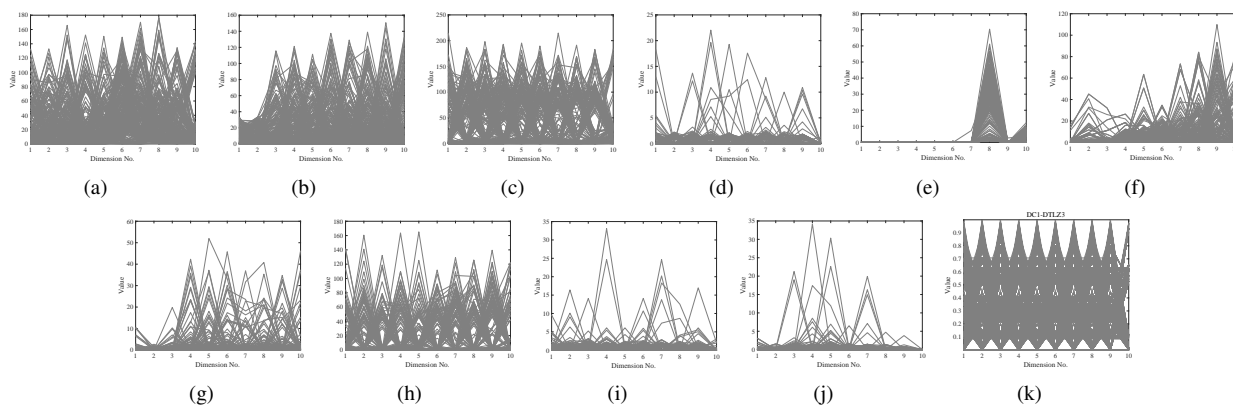


Fig. S-7. Feasible and non-dominated solutions with the median HV value among 30 runs on 10-objective DC1-DTLZ3. (a) NSGA-II-ToR, (b) ToP, (c) CCMO, (d) NSGA-III, (e) I-DBEA, (f) PPS, (g) C-TAEA, (h) TiGE-2, (i) DCNSGA-III, (j) CMME, and (k) true PF.

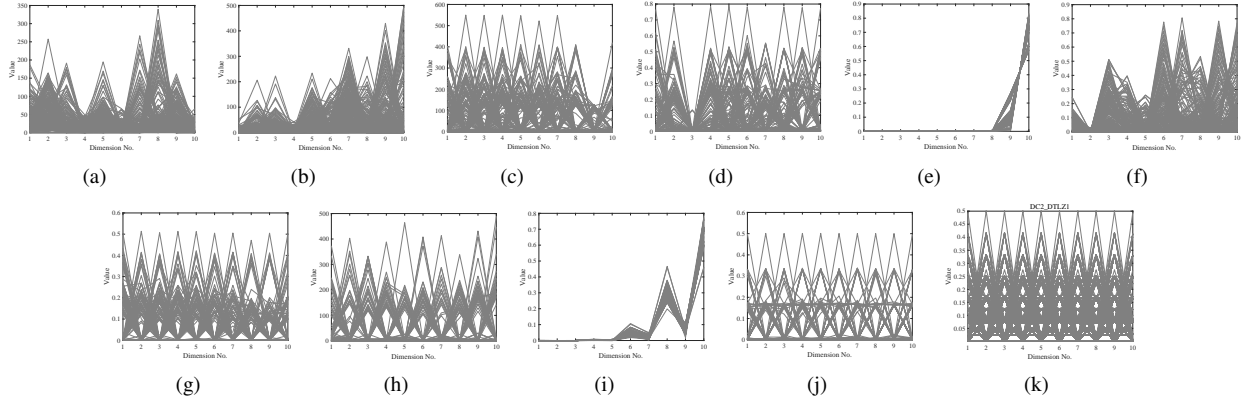


Fig. S-8. Feasible and non-dominated solutions with the median HV value among 30 runs on 10-objective DC2-DTLZ1. (a) NSGA-II-ToR, (b) ToP, (c) CCMO, (d) NSGA-III, (e) I-DBEA, (f) PPS, (g) C-TAEA, (h) TiGE-2, (i) DCNSGA-III, (j) CMME, and (k) true PF.

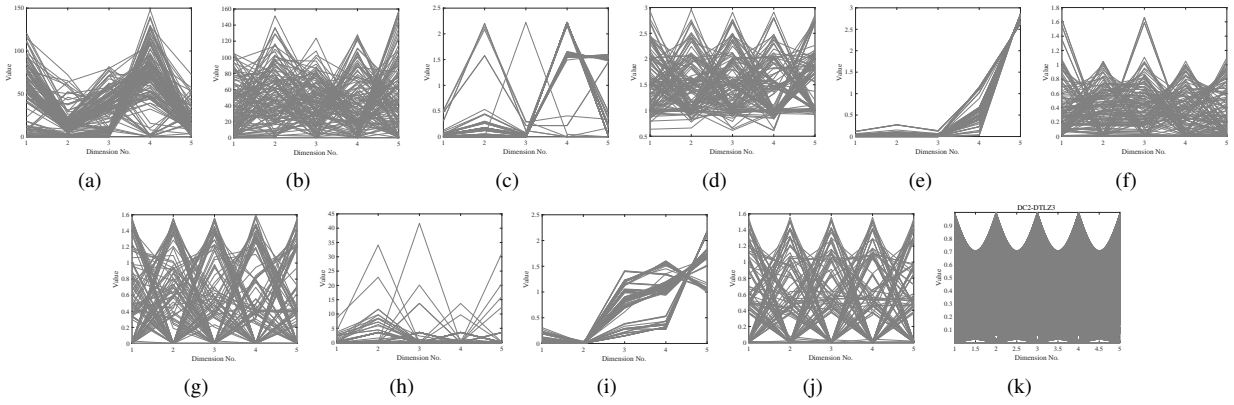


Fig. S-9. Feasible and non-dominated solutions with the median HV value among 30 runs on 5-objective DC2-DTLZ3. (a) NSGA-II-ToR, (b) ToP, (c) CCMO, (d) NSGA-III, (e) I-DBEA, (f) PPS, (g) C-TAEA, (h) TiGE-2, (i) DCNSGA-III, (j) CMME, and (k) true PF.

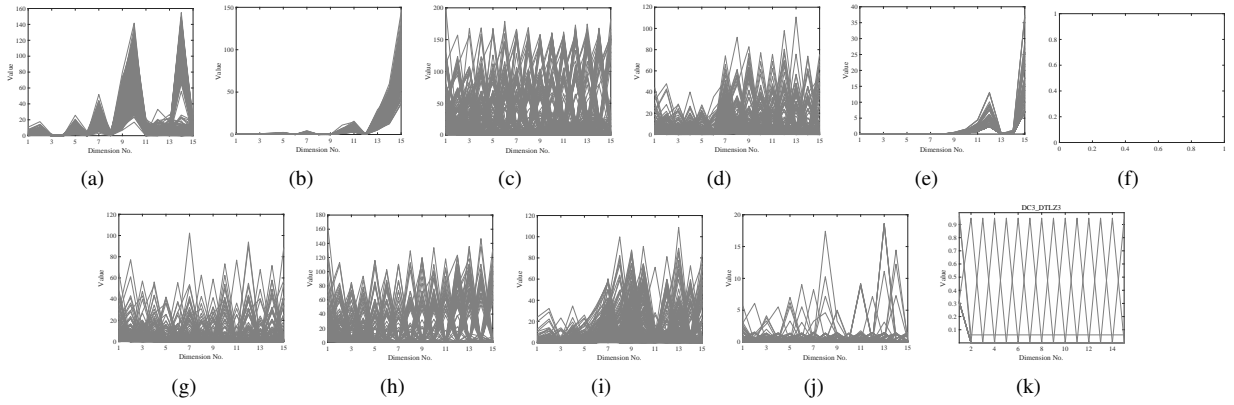


Fig. S-10. Feasible and non-dominated solutions with the median HV value among 30 runs on 15-objective DC3-DTLZ3. (a) NSGA-II-ToR, (b) ToP, (c) CCMO, (d) NSGA-III, (e) I-DBEA, (f) PPS, (g) C-TAEA, (h) TiGE-2, (i) DCNSGA-III, (j) CMME, and (k) true PF.

TABLE S-IX
PARAMETER SETTINGS FOR THE 10 METHODS ON REAL-WORLD APPLICATION PROBLEMS.

Problem	m	n	N	E_{\max}
Gear Box Design	3	7	91	26, 250
Multi-product Batch Plant	3	10	91	26, 250
Heat Exchanger Network Design	3	9	91	26, 250

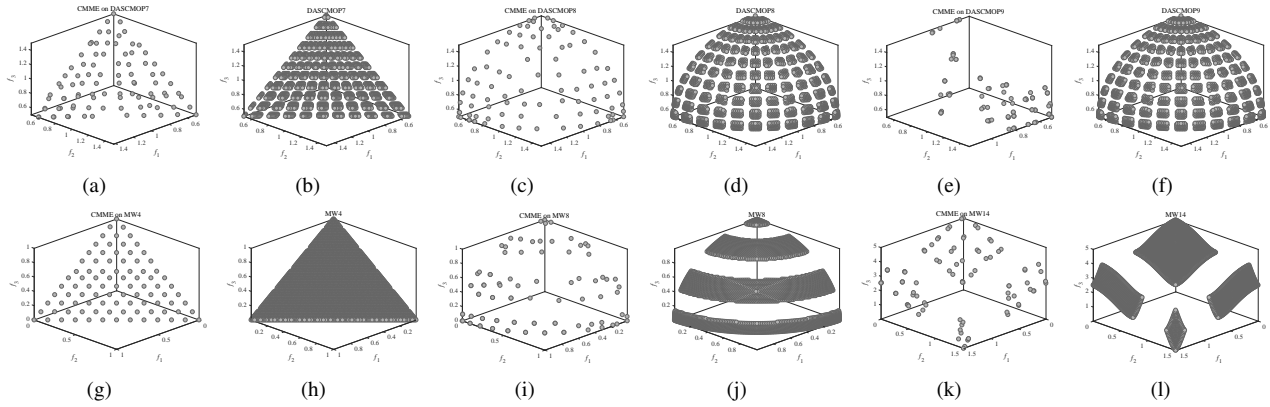


Fig. S-11. Feasible and non-dominated solutions with the median HV value among 30 runs on 3-objective DAS-CMOP and MW instances obtained by CMME.

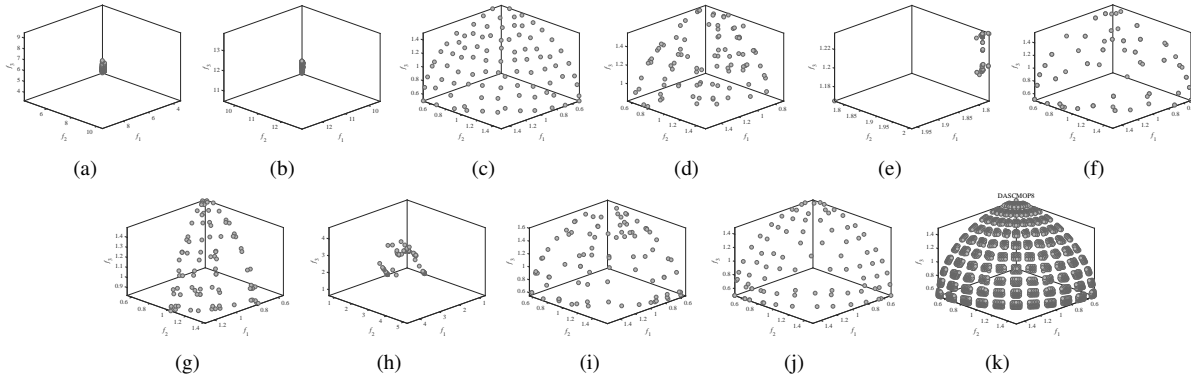


Fig. S-12. Feasible and non-dominated solutions with the median HV value among 30 runs on 3-objective DAS-CMOP8. (a) NSGA-II-ToR, (b) ToP, (c) CCMO, (d) NSGA-III, (e) I-DBEA, (f) PPS, (g) C-TAEA, (h) TiGE-2, (i) DCNSGA-III, (j) CMME, and (k) true PF.

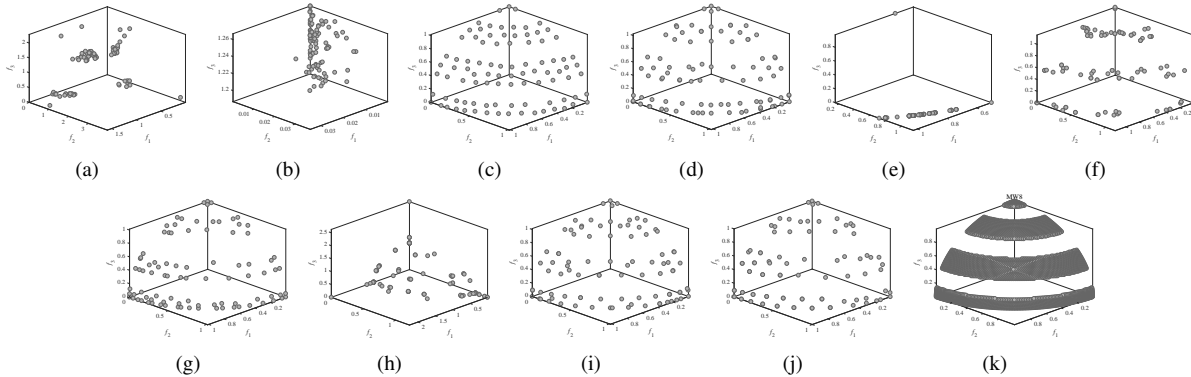


Fig. S-13. Feasible and non-dominated solutions with the median HV value among 30 runs on 3-objective MW8. (a) NSGA-II-ToR, (b) ToP, (c) CCMO, (d) NSGA-III, (e) I-DBEA, (f) PPS, (g) C-TAEA, (h) TiGE-2, (i) DCNSGA-III, (j) CMME, and (k) true PF.

TABLE S-X
HV OF CMME AND CMOEAS/CMAOEAS IN COMPARISON ON RW-MOP.

Problem	NSGA-II-ToR	ToP	CCMO	NSGA-III	I-DBEA	PPS	C-TAEA	TiGE-2	DCNSGA-III	CMME
Gear Box Design	8.7162e-2 (2.80e-3)	8.9136e-2 (2.03e-4)	8.8747e-2 (2.29e-4)	9.0018e-2 (2.67e-4)	1.8666e-2 (1.89e-2)	8.9439e-2 (2.81e-4)	8.7481e-2 (3.22e-3)	8.6687e-2 (8.52e-4)	7.9700e-2 (1.26e-2)	8.9287e-2 (1.19e-3)
Multi-product Batch Plant	2.2464e-1 (1.87e-2)	3.0167e-1 (6.31e-2)	3.1838e-1 (1.09e-2)	3.2279e-1 (1.39e-2)	1.6397e-1 (6.54e-2)	3.0697e-1 (3.05e-2)	1.3558e-1 (6.86e-2)	3.1729e-1 (1.66e-2)	3.0150e-1 (4.03e-2)	3.2988e-1 (1.27e-2)
Heat Exchanger Network Design	NaN (NaN)	NaN (NaN)	NaN (NaN)	NaN (NaN)	NaN (NaN)	7.7253e-1 (2.98e-1)	NaN (NaN)	1.0034e+0 (4.74e-3)	NaN (NaN)	1.0083e+0 (1.43e-2)
+/-/≈	0/30	0/21	0/30	1/20	0/30	0/21	0/30	0/21	0/30	

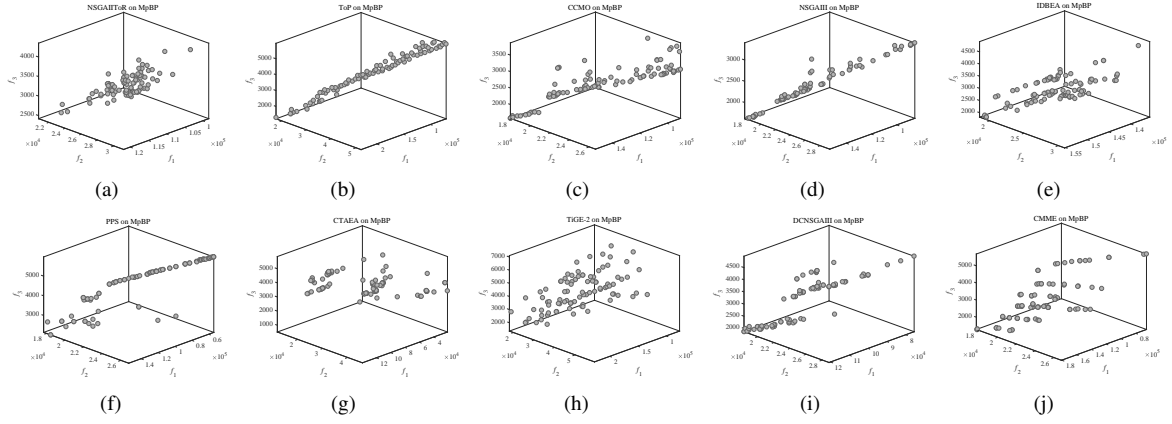


Fig. S-14. Feasible and non-dominated solutions with the median HV value among 30 runs on Multi-product Batch Plant problem. (a) NSGA-II-ToR, (b) ToP, (c) CCMO, (d) NSGA-III, (e) I-DBEA, (f) PPS, (g) C-TAEA, (h) TiGE-2, (i) DCNSGA-III and (j) CMME.